Abstract—Frequency protection is an important part of the art of power system relaying. On one hand it covers rotating machines and other frequency-sensitive apparatus from potential damage or extensive wear. On the other hand it is a part of load shedding schemes protecting the system. Unlike many other protection signals, power system frequency is not an instantaneous value. Moreover, there is no unambiguous definition of power system frequency assuming system transients and multi-machine systems. This paper discusses the power system frequency definition, signal models for frequency measurement, frequency measurement algorithms and fundamentals of frequency relaying. First, the concept of power system frequency is discussed and clarified in the context of a large scale system. The mathematical expressions of the system frequency and instantaneous frequency are presented on the basis of different signal models. A summary of the requirements for frequency measurement in applications such as under/over frequency relays, synchrocheck relays, phasor measurement units is given. Second, typical frequency measurement methods are reviewed and the performance evaluation are discussed. Third, simulation tests of the typical algorithms are provided to demonstrate various aspects of the frequency measurement, including metering accuracy, time response, tracking capability, and performance under noisy / harmonics conditions. In the end, some practical aspects in designing and testing the frequency relays are discussed.

I. INTRODUCTION

Frequency is an important parameter in power system to indicate the dynamic balance between power generation and power consumption. The system frequency and its rate-of-change are used directly for generator protection and system protection. When there are disturbances or significant load variations, the under/over-frequency relays could trip the units to avoid damage to the generators. When the system is about to lose its stability, the under-frequency relays can help to shed off non-critical loads so that the system balance could be restored. Power system stability can also be improved by installing power system stabilizers (PSS). A PSS could use the frequency of voltage signal taken at the generator terminal to derive the rotor speed, so that the excitation field and the power output of a generator can be adjusted by a feedback control scheme. In addition to direct usage in protection and control schemes, the function of frequency tracking is an indispensable part of modern digital relays because many numerical algorithms are sensitive to the variation of fundamental frequency. For example, the digital Fourier Transform (DFT) is widely used to compute phasors of voltage and current signals. If the sampling frequency is not the assumed multiple of signal frequency, leakage error would occur in phasor estimation. Without proper compensation, the overall performance of the protective relay will be impacted. Highly accurate and stable frequency measurement is always desirable for power system applications. However, the dynamic characteristic of signals in power system have brought challenge in designing a frequency estimation algorithm that is accurate, fast and stable under all kinds of conditions. To tackle this problem, researchers have proposed many numerical algorithms, such as zero crossing, DFT with compensation, phase locked loop, orthogonal decomposition, signal demodulation, Newton method, Kalman filter, neutral network etc. This paper provides a review on the concept of power system frequency, the frequency measurement algorithms and some fundamental aspects related to frequency relaying. The paper is organized as follows: The concept of frequency is discussed in section II. Section III and IV describe the signal models and the requirement for frequency relaying. Frequency measurement algorithms and the performance evaluation are reviewed in section V and VI. Section VII presents some simulation test results with respect to four selected algorithms. The frequency relay design and test are discussed in section VIII. Summary is given in the end.

II. THE CONCEPT OF FREQUENCY

A. The general definition and instantaneous frequency

The general definition of frequency in physics is the number of cycles or alternations per unit time of a wave or oscillation. Assuming a signal has N cycles within a period of Δt, its frequency will be

$$f = \frac{N}{\Delta t}. \quad (1)$$

From this general definition, one can derive that the signal needs to be periodical and the frequency is not an instantaneous quantity. However, it is common that frequency is used to characterize arbitrary signals including aperiodic signals. Meanwhile, the term of instantaneous frequency is seen from time to time in the literature. As a matter of fact, many frequency estimation algorithms are based on the concept of instantaneous frequency. These paradoxes can be resolved by extending the definition of frequency.

Using Fourier transform, an arbitrary signal can be decomposed into a weighted sum of periodic components in the form of sine / cosine waves. Let the signal be s(t) in time domain, its frequency domain correspondence is

$$S(f) = \int_{-\infty}^{\infty} s(t)e^{-j2\pi ft} dt. \quad (2)$$
where a particular \( S(f_0) \) gives the amplitude of the component that has a frequency \( f_0 \). If the signal is strictly periodic, it has one fundamental frequency. If the signal is aperiodic, it has multiple frequencies or even infinite number of frequencies. The frequency of each sinusoidal component follows the general definition. This way, the frequency definition is extended for aperiodic signals.

For a sinusoid signal in the form of \( s(t) = A \sin(\varphi) \), it can be viewed as the projection of a rotating phasor to the imaginary axis of a complex plane. The angular speed of the phasor is \( \frac{d\varphi}{dt} \). As a rotating phasor, the recurrence of the signal value means that \( \varphi \) is increased by \( 2\pi \). Because of this, the phasor repeats \( \frac{2\pi}{2\pi} \) times during \( \Delta t \) and the frequency is \( \frac{1}{\Delta t} \) according to the frequency definition in Eq. (1). Taking the limit \( \Delta t \to 0 \), the instantaneous frequency is defined as

\[
f(t) = \frac{1}{2\pi} \frac{d\varphi}{dt}.
\]

The above two extensions of frequency definition have played important roles in the area of signal processing. The Fourier transform in Eq. (2) is meaningful for stationary signals that the spectrum are constant in a window of time. For a non-stationary signal that the spectrum are time-varying, the instantaneous frequency can be used to characterize it. However, the concept of instantaneous frequency is controversial and application related. For example, a complex signal \( s(t) \) with following form

\[
s(t) = A(t) e^{j\varphi(t)}
\]

has both amplitude \( A(t) \) and phase \( \varphi(t) \) that are time-varying. When the signal is to be reconstructed from the sample values, it could be written either in amplitude-modulation form

\[
s(t) = A(t) e^{j\omega_0 t}, \tag{3}
\]

or phase-modulation form

\[
s(t) = A_0 e^{j\varphi(t)}, \tag{4}
\]

where \( \omega_0 \) and \( A_0 \) are constants. The instantaneous frequencies corresponding to Eq. (3) and Eq. (4) would be completely different. It shows that the instantaneous frequency needs to be defined in the context of a specific application.

**B. Frequencies for power system**

In power system, the voltage or current signals for frequency measurement are originated from the synchronous machines (generators) whose rotating speed are proportional to the frequency of the generated voltage. The mechanical frequency of a generator is its rotor speed

\[
f_m = \frac{1}{2\pi} \frac{d\theta_m}{dt},
\]

where \( \theta_m \) is the spatial angle of the rotor. The frequency of the generated voltage is

\[
f_e = \frac{1}{2\pi} \frac{d\theta_e}{dt},
\]

where \( \theta_e \) is the electrical angle that is proportional to \( \theta_m \) of a n-pole machine. From these equations, the frequency has clear physical meaning for a stand-alone generator. Both general definition of frequency and the extension of instantaneous frequency fit well in this case.

It is natural to extend the instantaneous frequency notation of the generator internal voltage to any nodes in the system. Using rotating phasor \( \vec{v}(i,t) \) to represent node voltage, the instantaneous frequency of the \( i^{th} \) node in the system can be defined as the phasor rotating speed,

\[
f(i,t) = \frac{1}{2\pi} \frac{d}{dt} \tan^{-1}\left( \frac{\text{Im}(\vec{v}(i,t))}{\text{Re}(\vec{v}(i,t))} \right)
\]

where \( \text{Im}(\vec{v}(i,t)) \) and \( \text{Re}(\vec{v}(i,t)) \) is the phasor rotating angle of the voltage signal on the complex plane. The frequency for current signal has the same expression. In power system, it is more meaningful to use a single quantity to represent the three phase signals. [12] proposed to use a composite space phasor derived from \( \alpha / \beta \) transform to represent the three-phase signal. The composite phasor is actually the scaled positive sequence component.

\[
\vec{v}_p = \frac{1}{\sqrt{3}}(v_1(t) + \alpha v_2(t) + \alpha^2 v_3(t))
\]

where \( \alpha = e^{j2\pi/3} \). The frequency is still defined as the rotating speed of phasor \( \vec{v}_p \) as in Eq. (5). Using positive sequence component, not only all three phases can be handled at the same time, the error from 3rd harmonics, dc component etc. for frequency estimation is also reduced.

For frequency relaying in most cases, the system frequency is the target as it is used to reflect the power balance of the system or a region. Since the frequency is obtained from each individual node, a question arises: can the measured frequency be taken as system frequency? The answer is yes and no, depending on the system condition, the application and the frequency estimation method.

In a power system, if the power generation and consumption are perfectly balanced and all the generators are in synchronism, the frequency of any node can be taken as system frequency. However, a power network is such a dynamic system that unbalance between generation and load always exists. Especially, when there is a disturbance such as fault on a critical transmission line or loss of a large generating unit, the balance between the generation and the load would be temporarily disturbed. Consequently, the power balance at each individual generating unit would be different. From [46], the swing equation of the \( i^{th} \) generator in a multi-machine system is

\[
M_i \frac{d\Delta \omega_i}{dt} = P_{mi} - P_{ei} - P_{di}.
\]

where \( M_i \) is the inertia coefficient of the \( i^{th} \) machine, \( P_{mi} \), \( P_{ei} \) and \( P_{di} \) are the mechanical power, electrical power and damping power respectively. This equation tells that rotor is accelerated or decelerated by the power unbalance \( (P_{mi} - P_{ei}) \) and the power \( P_{di} \) absorbed by the damping forces. The corresponding frequency would differ from generator to generator. During the electromechanical dynamics in the
system, a generator that is close to the disturbance will have an instantaneous rotor speed variation in response to the disturbance. But for the generators far away, the rotor speed and mechanical power output would not change at the first instant. The frequency difference will cause electromechanical wave propagation in the network to produce different frequency dynamics at different nodes in the system. From the simulation test in [63], the speed of the frequency wave propagation is 400-600 miles/sec for a 1800MW loss in Eastern US system.

Therefore, the node frequency is a local quantity that may not fully represent the system frequency, which is a global value that can be defined as the weighted average of the node frequencies or the equivalent frequency at the center of inertia [70].

$$f_s = \frac{\sum_{i=1}^{N} H_i f_i}{\sum_{i=1}^{N} H_i}$$  \hspace{1cm} (6)

where \(H_i\) is the inertia constant of the \(i\)th generator or the equivalent generator of a region. The averaging process in this equation should be carried out over all locations for a fixed time window to yield the system frequency. Nowadays this has become possible by utilizing a group of GPS-synchronized PMUs that are connected through high speed communication network. However, for practical reason, the system frequency is usually approximated by the time averaging of individual node frequency,

$$f_s \approx f_i = \frac{1}{t-t_0} \int_{t_0}^{t} f(i,t) dt.$$  \hspace{1cm} (7)

From Eq. (6) and Eq. (7), the system frequency is not an instantaneous value. However, the concept of instantaneous frequency can still be used in some frequency estimation algorithms. The average value of the estimated frequency can be used to approximate system frequency. This leads to another issue: the frequency results from different intelligent electronic devices (IEDs) could be different. Some IEDs are based on the periodic characteristic of the signal, some are based on the concept of instantaneous frequency. Different algorithms also have different accuracy and different response to harmonics, noise, time-varying amplitude, etc. Therefore, the measured frequency at a node in the system should be called apparent frequency, which is a reflection of the actual node frequency in the IEDs. For most IEDs, it is usually a window of sample values that are used to compute the frequency and the results are usually smoothed by moving average filters. This way, the apparent frequency would be close to node frequency and system frequency.

In brief, the node frequency, generator frequency, system frequency and apparent frequency are different quantities, even though their value could be very close, particularly under very slow system disturbances and in steady states. To understand the difference would be helpful in design and test of frequency-related applications in power system.

III. SIGNAL MODELS FOR FREQUENCY MEASUREMENT

The modeling of signals is the first step to the frequency measurement problem. As a mathematical description of signal, a model would establish the relationship between the unknown parameters and the observed sample values. The signal models that are commonly used for frequency measurement are summarized in this section.

A. Basic signal model

The most widely used signal model in power system is a voltage signal expressed by

$$v(t) = A \cos(\omega t + \varphi).$$  \hspace{1cm} (8)

where \(A\) is the amplitude, \(\omega\) is the angular frequency and \(\varphi\) is the phase angle. For a stationary signal, the frequency is simply \(\omega/2\pi\). For a non-stationary signal, the frequency and phase angle can not be considered separately from this model. Some algorithms would take \(\omega\) and \(\varphi\) as two variables but estimate them simultaneously; Some would use a fixed value for \(\varphi\) and leave only \(\omega\) as the only variable within the cosine function; Some algorithms would take \(\omega\) as nominal frequency and compute the frequency deviation from the phase angle variation

$$f = f_0 + \frac{1}{2\pi} \frac{d\varphi}{dt},$$  \hspace{1cm} (9)

which is in line with the expression of instantaneous frequency.

B. Signals models with harmonics, noise and decaying dc component

Inevitably, the voltage or current signal in a power system could be contaminated by harmonics, noise and dc component. Some frequency measurement algorithms assume those should be handled by separate filters. Some just include them in the signal model

$$v(t) = A_0 e^{-t/\tau} + \sum_{k=1}^{M} A_k(t) \sin(\omega_k t + \varphi_k(t)) + \varepsilon,$$

where \(A_0\) is the initial amplitude of the dc component that has time constant \(\tau\), \(A_k(t)\) represents the amplitude of the \(k\)-th harmonic, \(\varepsilon\) is noise and \(M\) gives the maximum order of the harmonics. From this model, the frequency deviation can be regarded as phase angle change of fundamental component

$$f = f_0 + \frac{1}{2\pi} \frac{d\varphi_1(t)}{dt},$$

where \(\varphi_1(t)\) represents the phase angle of fundamental component that is slightly different from Eq. (9).

C. Complex signal model from Clarke transformation

In a power system, it is meaningful to measure the frequency for all three phases simultaneously. Instead of combining the measuring results from three single-phase
voltages, the Clarke transformation is used to modify the three-phase system to a two-phase orthogonal system,

\[
\begin{bmatrix}
v_\alpha \\
v_\beta
\end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} - \frac{j}{2} & -\frac{1}{2} + \frac{j}{2} \\ 0 & \frac{1}{2} + \frac{j}{2} & \frac{1}{2} - \frac{j}{2} \end{bmatrix}
\begin{bmatrix}
v_\alpha \\
v_\beta
\end{bmatrix}
\]

From the above equation, either \(v_\alpha\) can be used alone for frequency measurement, or a composite signal \(v = v_\alpha + jv_\beta\) can be used. Using Clarke transformation, not only the three-phase signals could be considered at the same time, the composite signal in complex form could also be useful in some algorithms such as [2], [6], [48], [57]. The model is also less susceptible to harmonics and noise. The disadvantage is that three-phase unbalance could impact the related algorithms.

D. Signal model using positive sequence component

In [11], [47], [57], the positive sequence voltage is used for frequency estimation. The positive sequence component has the same advantage as the composite signal from \(\alpha \beta\) transform. Actually, they are equivalent under system balance condition. There are various solutions to compute the sequence components from sample values. Since the positive sequence voltage \(V_\alpha\) is a space vector rotating with angular speed \(2\pi f\), the frequency can be computed by

\[
f = \frac{1}{2\pi} \frac{d}{dt} \text{arg}(V_\alpha).
\]

IV. FREQUENCY RELAYING AND THE REQUIREMENTS

The main applications of frequency relaying include under / over frequency relays for generator protection or load shedding schemes, the voltage / frequency (V/Hz) relays for generator/transformer overexcitation protection, synchrocheck relays, synchrophasors and any phasor-based relays that incorporating frequency tracking mechanism for accurate phasor estimation.

When there is an excess of load over the available generation in the system, the frequency drops as the generators would slow down in attempt to carry more load. If the underfrequency or overfrequency condition lasts long enough, the resulted thermal stress and vibration could damage the generators. If the load / generation unbalance is severe, the generator shall be tripped by its unit protection, which could consequently worsen the system unbalance condition and lead to a cascading effect of power loss and system collapse. On one hand, the underfrequency relays can be used to trip the generators when the system frequency is close to the withstand limits of the units. On the other hand, the underfrequency relays can be used to automatically shed some pre-determined load so that the load / generation balance could be restored. Such load shedding action must be taken promptly so that the remainder of the system could recover without sacrificing the essential load. Most importantly, the action shall be fast enough to prevent the cascading of generation loss into a major system outage. For this type of applications, the frequency relays should have high accuracy because as little as 0.01Hz of frequency deviation could represent tens of megawatts in power unbalance. It is generally required that a frequency relay has a 1mHz resolution. Meanwhile, the frequency measurement must be stable and robust under various conditions.

When a generator unit is under AVR control at reduced frequency during unit start-up and shutdown, or under over-voltage conditions, the magnetic core of the generator or transformer could saturate and consequent excessive eddy currents could damage the insulation of the generator / transformer. To prevent this, relays based on volts / hertz measurement can be deployed to detect this over-excitation condition. The accuracy requirement is the same as the underfrequency relay.

The synchrocheck relays are used to supervise the connection of two parts of a system through the closure of a circuit breaker. The difference of frequency, phase angle and voltage need to be within the setting range to prevent power swings or excessive mechanical torques on the rotating machines. In general, a setting of around 0.05Hz is sufficient and the frequency resolution of a synchro-check relay could be in the range of 10mHz.

For microprocessor-based relays, the frequency tracking mechanism is critical to phasor estimation. Frequency tracking usually indicates that a digital relay can adjust its sampling frequency according to the signal frequency, in order to reduce the phasor estimation error. Most digital relays use phasor estimation as the foundation of protection functions since phasors can help to transform the differential equations of electrical circuits into simple algebra equations. Though the expression of a phasor is independent of frequency, different signal frequency could result in different phasors. Without frequency tracking, the performance of the protection functions will be impaired under off-nominal frequency conditions. For phasor estimation, the frequency tracking shall be as fast as possible to follow the frequency variations, under the condition that stability of the frequency measurement is maintained. In addition, the range of frequency tracking for generator protection needs to be wide enough to cope with the generator starting-up and shutting-down.

The frequency and frequency rate-of-change are also integrated part of synchrophasor units for wide area protection and control. In 2003, an internet-based frequency monitoring network (FNET) was set up in U.S. to make synchronized measurement of frequency for a wide area power network [37], [71]. From the synchronized data collected by FNET, significant system events such as generator tripping can be located by event localization algorithms [15] that are based on the traveling speed of the frequency perturbation wave and the distance between observations points. For this type of applications, the accuracy of frequency metering should be as high as possible since minor frequency error could mean hundreds of miles difference in fault localization.

In general, the frequency measurement should have enough accuracy and good speed. A ±1mHz accuracy is deemed good enough for most frequency relaying applications. However, the ±1mHz accuracy is only valid when the frequency has slow changes. Though fast frequency
estimation could mean less dynamic error, the accuracy and the speed requirement are mutually exclusive at a certain point. More error could be produced in pursuit of fast frequency estimation, especially when the system or signals are under adverse conditions. In power system, the voltage or current signal for frequency estimation could be contaminated by harmonics, random noise, CT saturation, CVT transients, switching operation, disturbance, electromagnetic interference, etc. It is imperative that a frequency relay shall not give erroneous results to cause false relay operation. To summarize, the following criteria need to be satisfied for frequency relaying:

- The measured frequency or frequency rate-of-change should be the true reflections of the power system state;
- The accuracy of frequency measurement should be good enough under system steady state and dynamic conditions;
- The frequency estimation should be fast enough to follow the actual frequency change, in order to satisfy the need of the intended application;
- The frequency tracking for generator protection should have wide range to handle generator starting-up and shutting-down;
- The frequency measurement should be stable and robust when the signal is distorted.

V. FREQUENCY MEASUREMENT ALGORITHMS

In the past, a solid state frequency relay can use pulse counting between zero-crossings of the signal to measure the frequency. The accuracy could be as high as ±1 ~ 2mHz [43] under good signal conditions, but the relay is susceptible to harmonics, noise, dc components, etc. Nowadays, with the prevalence of microprocessor-based relays and cheaper computational power, many numerical methods for frequency measurement were applied or proposed, including:

- Modified zero-crossing methods [4], [3], [44], [52], [53]
- DFT with compensation [23], [28], [65], [68]
- Orthogonal decomposition [40], [55], [59]
- Signal demodulation [2], [11]
- Phase locked loop [12], [16], [27]
- Least square optimization [7], [34], [49], [62]
- Artificial intelligence [8], [13], [30], [32], [45], [58]
- Wavelet transform [31], [36], [35], [9], [64]
- Quadratic forms [29], [30]
- Prony method [38], [42]
- Taylor approximation [51]
- Numerical analysis [67]

Some of these methods are briefly reviewed in this section.

A. Zero-Crossing

The zero-crossing (ZC) is the mostly adopted method because of its simplicity. From the frequency definition, the frequency of a periodic signal can be measured from the zero-crossings and the time intervals between them. A solid state frequency relay could detect the zero-crossings by using voltage comparators and a reference signal. In a software implementation, the zero-crossing can be detected by checking the signs of adjacent sample values. The duration between two zero-crossings could be obtained from the sample counts and the sampling interval. Fig. 1 shows the zero-crossing detection using digital method. The accuracy of ZC could be influenced by zero-crossing localization, quantization error, harmonics, noise and signal distortion. The quantization error is negligible if high sampling rate and high precision A/D converter are used. A lowpass filter can be applied to reduce the harmonics and noise in the signal. The random error caused by zero-crossing localization on time axis could be significant if the sampling frequency is not high enough. [4] proposed to use polynomial curve to fit the neighboring samples of the zero-crossing. The roots of the polynomial can be solved by least error squares (LES) method and one of the roots is taken as the precise zero-crossing on time axis. The disadvantage of this method is the high computational cost for curve fitting and polynomial solving. In practice, the linear interpolation is used mostly, as illustrated in Fig. 1. To improve the accuracy of ZC, a post-filter such as a moving average filter is usually applied.

The slow response to frequency change is another issue for ZC since the measured frequency can be updated after at least half a cycle. In practice, it takes a few cycles to obtain good accuracy. Including the delay brought by the pre-filters and post-filters, the total latency could be significant. A level crossing method was proposed in [44] to supplement the ZC by multiple computations of the periods between non-zero voltage level crossings. It makes use of all the sample values to improve the dynamic response of the algorithm.

But the method is susceptible to amplitude variations and signal distortion. In [1], a three-point method is used to supplement the ZC. The frequency can be quickly derived from three consecutive samples. However, the method is highly susceptible to noise, harmonics and amplitude variations.

In brief, a zero-crossing method has its advantage of simplicity. But it needs to be supplemented with other techniques to obtain good accuracy and good dynamic response. In some cases, the overall algorithm becomes so complicated that the simplicity of the zero-crossing method has been lost.

B. Digital Fourier Transform

The digital Fourier Transform (DFT) is widely used for voltage and current phasors calculation. For a discrete signal
v(k), if the DFT data window contains exactly one cycle of samples, the phasor of fundamental frequency is given by,

\[ V_k = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} v(k + n - N + 1)e^{-j2\pi n/N} \] (10)

where \( N \) is the number of samples and the subscript \( k \) represents the last sample index in the data window. The resulted phasor rotates on the complex plane with an angular speed determined by signal frequency, which can be taken as instantaneous frequency

\[ f = \frac{1}{2\pi} \arg[V_{k+1}] - \arg[V_k]. \]

where \( \arg[V_k] = \tan^{-1}\{\text{Im}[V_k]/\text{Re}[V_k]\} \). The phasor estimation and frequency estimation are highly correlated to each other. If the design assumed that sampling rate is an integer multiple of signal frequency, DFT will produce leakage error on both phasor and frequency measurement for the signal with off-nominal frequency. Using DFT, a N-point data sequence in time domain will produce N discrete frequency bins in frequency domain. If the signal frequency is not overlapping any of these frequency bins, the 'energy' from the samples will leak to the neighboring bins. The closest frequency bin that is used to approximate the signal frequency will get the most 'energy'. Hence, the leakage error is introduced into the estimated phasor and frequency. Fig. 2-(a),(b) present the frequency domains of a 60Hz signal and a 59Hz signal as the DFT results under the sampling rate 3840Hz. The corresponding phasors out of DFT are shown in Fig. 2-(c),(d). Without compensation, the magnitude and angle for the 59Hz signal oscillate and deviate from the actual values. In contrast, the magnitude and angle for the 60Hz signal are straight lines. Almost all DFT-based frequency measurement algorithms are focusing on how to reduce or eliminate leakage error. There are four main approaches:

1) The length of data window is fixed, the sampling frequency is updated by the estimated signal frequency [5];

2) The sampling frequency is fixed, the length of data window is updated by the estimated signal frequency [14], [23];

3) The length of data window is fixed, the data are re-sampled to ensure one cycle of data in the window [28];

4) Both the sampling frequency and the length of data window are fixed, the leakage error is compensated analytically [4], [47], [65], [69].

The first three approaches are based on the fact that leakage error can be canceled out if the sampling frequency is an integer multiple of signal frequency, or equivalently, the DFT data window contains exactly \( n(n = 1, 2, \ldots) \) cycles of samples. Under this condition, the signal frequency will overlap one of the frequency bins in frequency domain so that no leakage would occur.

In [5], the variable-rate measurement is proposed for frequency measurement. A feedback loop is applied to adjust the sampling frequency until the derived frequency is locked with the actual signal frequency. Similar to a phase locked loop, this type of methods can achieve high accuracy since the feedback loop can force the error towards zero. However, the feedback could slow down the frequency tracking speed for a real-time application. With proper hardware and software co-design, this method is suitable for on-line frequency measurement.

In [23], the DFT data window has a variable length according to the estimated frequency, so that a cycle of samples could be included in the data window. Since the sampling frequency is fixed while the signal frequency is uncertain, it is not guaranteed that the updated data window would contain one cycle of samples exactly. Therefore, the leakage error cannot be eliminated by this method. For further compensation, [23] proposed to use the line-to-line voltage or positive sequence voltage to reduce the influence of harmonics and to use a moving average filter to smooth the estimation results. This method is easy to implement and the measurement range is wide, which is good for generator protection. However, the accuracy is limited because of the incomplete leakage compensation.

In [28], the hardware samples are re-calculated into software samples so that the data window will always include a fixed amount of samples for one cycle of signal exactly. A feedback loop is used to adjust the re-sampling by the estimated frequency until the error is lower than a threshold. This method is more accurate than [23] and simpler than [5]. Again, the feedback loop needs careful design for good dynamic response in real-time applications.

In [65], [68], a number of successive phasors out of DFT are utilized to cancel the leakage error without changing the sampling rate and the data window length. The method in [68] does not make any approximations to cancel out the leakage error so that high accuracy can be achieved. The details of this algorithm is given in the Appendix. From the simulation tests, this method can achieve both high accuracy and good dynamic response, but it is susceptible to harmonics, noise and dc component.
In summary, DFT can be used to estimate fundamental frequency, phasor and harmonics simultaneously, which is its advantage over the other single-objective algorithms. However, the leakage effect could have significant impact on the phasor and frequency estimation. The DFT methods need to be supplemented by compensation techniques for good accuracy. Comparing various compensation techniques, the algorithms in [5], [28], [68] are recommended for phasor and frequency estimation.

C. Signal Decomposition

Like DFT, this group of methods will decompose the input signal into sub-components so that the problem is transformed and useful information can be retrieved. The approaches in [40], [55], [59] would decompose the input signal into two orthogonal components to derive the frequency after some mathematical manipulations.

In [40], the input signal is decomposed by a sine filter and a cosine filter,

\[ v_1(t) = A \sin(2\pi ft + \varphi), \]
\[ v_2(t) = A \cos(2\pi ft + \varphi). \]

After taking the time derivatives of these two signals, the frequency is computed by

\[ f = \frac{v_2(t)v_1'(t) - v_1(t)v_2'(t)}{2\pi(v_1^2(t) + v_2^2(t))}. \] (11)

Eq. (11) is accurate hypothetically. However, error could stem from the signal decomposition and the approximation of the derivatives. From the frequency response of the sine / cosine filters in Fig. 3, their filter gains are the same only at nominal frequency. Error will be introduced for frequency estimation due to different filter gains at off-nominal frequencies. In [40], a feedback loop is designed to adjust the filter gains. After adjustment, good accuracy can be achieved but only in a narrow range around nominal frequency. Instead of using sine and cosine filters, [55] proposed to use finite impulse response (FIR) filters designed by optimal methods. Different coefficients are used for different off-nominal frequencies. The coefficients with 1Hz step are calculated off-line and stored in a look-up table. For other frequencies, interpolation is performed on-line to adjust the coefficients. A feedback loop is applied to select the filters from the measured frequency. The accuracy is improved by the feedback adjustment. Meanwhile, the harmonics can be suppressed by the FIR filters. However, the convergence may be slow for a real-time application because of the feedback loop.

Without using feedback loop and orthogonal filters, [54] uses a group of FIR filters to derive the frequency. After pre-filtering, the input signal is decomposed by an all-pass filter and a low pass filter. The decomposed signals will then pass through two groups of cascading FIR filters. The frequency is then derived from the outputs of the two paths, during which the error brought by filter gains are canceled out. Compared with [40] and [55], there is no error compensation by a feedback loop and the filters are fixed so that no extra storage of coefficients is needed. However, the group delay of the FIR filters will slow down the dynamic response, and the frequency output is highly sensitive to harmonics and noises so that pre-filter design is critical to the overall performance of this method.

In [59], the impact of different filter gains are canceled out by a sequence of decomposed signals. After filtering, a new signal is produced by combining the sub-components. The historical values of this new signal are utilized to cancel the impact of filter gains. The details of this algorithm are given in the Appendix. Since the influence of unequal filter gains are completely canceled out, high accuracy can be achieved with this method. It is also simpler and faster in comparison with other decomposition algorithms. However, as it is based on the assumption that signal amplitude is stable for a window of data, the time-varying amplitude of a non-stationary signal could have impact on its accuracy.

D. Signal Demodulation

Instead of decomposing the input signal, a demodulation method starts from synthesizing a new signal. Fig. 4 illustrates the process of computing the frequency deviation by signal demodulation method (SDM). After the input signal is modulated by the reference signal that has nominal frequency, the resulted signal \( v_p \) contains a low frequency component and a near-double frequency component. Through the low-pass filter, the low frequency component \( v_c \) is retrieved and the frequency deviation is calculated as the rotating speed of \( v_c \). More details of this method are given in Appendix. The advantage of SDM is its simplicity and potential for high accuracy. However, the stopband attenuation of the low-pass filter must be high enough to remove the near-double frequency component. A compromise between the filter attenuation and filter delay must be made for the accuracy and the dynamic response of frequency measurement.

![Fig. 3. Frequency response of a sine filter and a cosine filter](image)

![Fig. 4. signal demodulation method](image)
E. Phase Locked Loop

A phase-locked loop (PLL) is a feedback system that responds to the frequency / phase change of the input signal by raising or lowering the frequency of a voltage controlled oscillator (VCO) until its frequency / phase matches the input signal. A typical PLL is composed by three parts as illustrated in Fig. 5. From the phase detector, a new signal \( v_p \) is produced from the input signal \( v_1 \) and the reference signal \( v_0 \). The synthesized signal \( v_p \) contains a low-frequency component corresponding to the frequency deviation. Passing a lowpass filter, the low frequency component \( v_c \) is retrieved and used as error signal to drive the voltage-controlled oscillator (VCO). The oscillation frequency of VCO is adjusted and the VCO output \( v_0 \) feeds back to the phase detector. The frequency difference of \( v_1 \) and \( v_0 \) will be smaller and smaller after each feedback until it is zero, which is the locked state of a PLL.

It is noted that a PLL for frequency measurement is quite similar to the signal demodulation method. Both use lowpass filters to demodulate the synthesized signal to get the frequency deviation. However, a PLL is characterized as a feedback system that frequency difference would be gradually reduced towards zero, which implied that a PLL could achieve very high accuracy on frequency measurement at the price of some time delay. Another advantage is that PLL is insensitive to harmonics and noise because of the lowpass filter and the feedback loop.

The critical part of a PLL design for frequency measurement is the phase detector. In [17], the transformed \( \alpha\beta \) signal is used as input of the phase detector. In [16], a proportional-integral (PI) controller is used to improve the performance and stability of the feedback system. In [18], [27], the phase detector consists of an in-phase component and a quadrature component to estimate the time derivative of the phase angle directly so that the nonlinear dependency of the error signal to the phase difference is avoided. With this design, the range of frequency measurement is wide and the convergence is claimed to be within a few cycles.

Because of its accuracy and robustness, a PLL can be applied in a line differential protection scheme for accurate data synchronization [41]. Combined with the GPS time, the system frequency is also utilized to synchronize the data packet that are exchanging continuously among the relays. For this type of applications, fast frequency tracking is not desired. Instead, the accurate and stable frequency measurement will help the data alignment for relays at different locations.

F. Non-linear iterative methods

A number of non-linear iterative methods were proposed for accurate frequency estimation, including: least error squares (LES) methods [19], [49], [50], [61], least mean squares (LMS) methods [25], [48], Newton methods [60], [62], Kalman filters [7], [10], [20], [21], [22], [26], steep descent method [34], etc. A common feature of these methods is to iteratively minimize the error between the model estimations and the sample values, so that parameters or states of the model could be derived.

1) Least Error Squares Method: [49] proposed the LES technique to estimate the frequency in a wide range. Using three-term Taylor expansion of Eq. (8) in the neighborhood of nominal frequency, the voltage signal is turned into a polynomial,

\[
v(t_1) = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 \quad (12)
\]

where \( v(t_1) \) is the sample value at time \( t_1 \), the coefficients \( a_{11..16} \) are known functions of \( t_1 \), the parameters \( x_1..x_6 \) are unknowns to be solved. E.g., \( x_1 = A \cos \varphi \) and \( x_2 = (\Delta f)A \cos \varphi \). Using \( m > 6 \) samples, a linear system with \( n = 6 \) unknowns is set up and the unknowns can be resolved by LES method. The frequency is obtained by \( f = f_0 + x_2/x_1 \) or \( f_0 + x_4/x_3 \). In this algorithm, the accuracy of frequency estimation is affected by the simplified signal model, the size of data window for LES, the sampling frequency and the truncation of the Taylor expansion. In addition, the matrix inversion that is used in every block calculation could bring numerical error in a real-time application. In order to improve the estimation accuracy of LES method, some error correction techniques [50], [61] were proposed. These techniques would increase the complexity of the algorithm while the accuracy may still be a problem.

2) Newton Type Methods: The Newton method in [60] takes the dc component, the frequency, the amplitude and the phase angle as unknown model parameters and estimate them simultaneously through an iterative process that aims at minimizing the error between the sample values and the model estimations. The updating step is derived from Taylor expansion and the steepest descent principle. The details of a Gauss-Newton algorithm are given in the Appendix. Using Newton methods, good accuracy can be achieved with moderate number of iterations. Meanwhile, the phasor is also obtained simultaneously. However, the algorithm may not converge if the initial estimation of the parameters are far from the actual values. The dynamic variations of both amplitude and frequency could also delay the convergence. To overcome these problems, the auxiliary methods such as ZC and DFT could be applied to initialize the frequency and amplitude and to supervise the convergence, as presented in the Appendix. Using supervised Gauss-Newton (SGN) method, not only the performance is improved, the frequency estimation is more robust under adverse signal conditions.
3) Least Mean Square Method: LMS is another type of iterative algorithm that uses a gradient factor to update the model parameters. The product of input and the estimation error is used to approximate the gradient factor for each iteration. In [48], the complex signal out of Clarke transform is used to estimate the frequency. The relationship between the current estimation \( \hat{v}_k \) and previous estimation \( \hat{v}_{k-1} \) is expressed as

\[
\hat{v}_k = e^{j\omega \Delta T} \hat{v}_{k-1} = w_{k-1} \hat{v}_{k-1}
\]

The variable \( w_k \) can be updated by

\[
w_{k+1} = w_k + \mu e_k \hat{v}_{k-1}^*,
\]

where \( e_k = v_k - \hat{v}_k \) is the error between sample value and the estimation, \( \hat{v}_{k-1}^* \) is the complex conjugate and \( \mu \) is the tuning parameter. When the error \( e_k \) is small enough, the frequency is derived from the variable \( w_k \).

\[
f(t) = \frac{1}{2\pi \Delta T} \sin^{-1} \text{Im}(w_k).
\]

Using LMS, both the accuracy and the frequency tracking speed can be satisfactory. In addition, as a curve fitting approach, the algorithm is insensitive to noise. However, the parameter \( \mu \) in Eq. (14) needs to be adjusted to accelerate the convergence. The main issue of the method in [48] is that complex model has to be used. As mentioned before, when there is three-phase unbalance, the complex signal out of Clarke Transform could cause error in frequency and phasor estimation.

4) Kalman Filters: Established on stochastic theory and state variable theory, a Kalman filter predicts the state and error covariance one step ahead from the historical observations, then the state estimation and error covariance are updated with the new observations. To apply Kalman filter for frequency estimation, the critical step is to establish a state difference equation and a measurement equation to relate the states and observations. The general expressions of these two equations are

\[
x_k = Ax_{k-1} + w_{k-1}, \quad z_k = Hx_k + v_k,
\]

where \( x_k \) is a vector of state variables, \( A \) represents the state transition matrix, \( z_k \) is the vector of current observations (the sample values), \( H \) is a relation matrix, \( w_k \) and \( v_k \) represent the process noise and measurement noise respectively. The state vector is recursively estimated by a Kalman filter equation

\[
\hat{x}_k = \hat{x}_{k-1} + K_k(z_k - H\hat{x}_{k-1}),
\]

where \( K_k \) is the Kalman gain that can be derived from a set of established procedures as described in [24], [66]. If the process model is non-linear, the extended Kalman Filter (EKF) that includes extra steps to linearize the models can be applied.

In [7], the state difference equation is established on the basis of a complex model,

\[
\begin{bmatrix}
    x_{1k} \\
    x_{2k} \\
    x_{3k}
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 & 0 \\
    0 & x_1(k-1) & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x_{1(k-1)} \\
    x_{2(k-1)} \\
    x_{3(k-1)}
\end{bmatrix},
\]

which includes three state variables

\[
x_1 = e^{j\omega T_s}, \quad x_2 = Ae^{j\omega kT_s+j\varphi}, \quad x_3 = Ae^{-j\omega kT_s-j\varphi}.
\]

The measurement equation is established as

\[
z_k = \begin{bmatrix} 0 & 0.5 & -0.5 \end{bmatrix} \begin{bmatrix} x_{1k} \\ x_{2k} \\ x_{3k} \end{bmatrix} + v_k.
\]

Using EKF procedures, the state variables can be estimated and updated. The frequency is derived from the state variable \( x_1 \).

A Kalman filter has the advantage of quick dynamic response and it can effectively suppress the white noise. However, the speed of convergence is up to the initial value of state variables, error covariance matrixes and noise covariance, which are set according to signal statistics. The accuracy is also influenced by the linearization and the simplification of the noise model. The computational expense of a Kalman filter is also considerable for a real-time application.

VI. PERFORMANCE EVALUATION

To evaluate the performance of a frequency relay or an frequency estimation method, three aspects should be considered: the accuracy, the estimation latency and the robustness. The maximum error, the average error and the estimation delay could be used as the performance indexes for a frequency relay. The maximum error is based on the momentary difference between the actual frequency and the estimated frequency. The average error is based on a window of data in which the average values are taken for both actual frequency and estimated frequency. The robustness is reflected by these indexes under adverse conditions. Some frequency relays claim ±1mHz resolution, which shall not be taken as the performance index. A frequency estimation method could be extremely accurate when the input signal is stable and clean, but highly inaccurate when the signal is distorted or contaminated by harmonics and noise. The accuracy should be obtained under adverse signal conditions to reflect the robustness of the relay. This also applies to the frequency estimation latency. Most frequency estimation methods use a window of data to derive the frequency, which would cause estimation delay when the frequency is time-varying. It is desirable that the latency should be as small as possible, under the restraints of accuracy requirement and robustness requirement. Because of the latency, the maximum error could be high while the average error is a better index to evaluate the relay or the algorithm. The data window length for average error can be different for different applications. For a underfrequency relay, a window of 5 – 10 cycles is sufficient to calculate the average error, since a time
delay of over 0.2s is usually set for the relay to make secure operation.

For evaluation purpose, some benchmark test signals can be used to get the maximum error and average error on the frequency measurement. The following conditions for setting up benchmark signals are proposed for the evaluation.

1) The frequency tracking range is 20 – 65Hz;
2) To simulate power swing, the signal frequency is modulated by a 1Hz swing, and the signal amplitude is modulated by a 1.5Hz swing;
3) The signal is contaminated by 3rd, 5th, 7th harmonics, the percentage is 5% each;
4) The signal contains dc component, the time constant could be set at 0.5;
5) The signal contains random noise with signal-to-noise ratio (SNR) 40dB;
6) The signal contains impulsive noise;
7) To simulate subsynchronous resonance, the signal contains 25Hz low frequency component;

Using individual condition or combined conditions, a number of analytical signals can be created to test different aspects of the frequency relay. In addition to analytical signals, the voltage or current signals obtained from transient simulation programs (such as EMTP, SIMULINK, RTDS, etc.) could be used to test the performance of a frequency relay. A good relay should have consistent performance indexes for various test signals.

VII. THE SIMULATION TESTS

In this section, four frequency estimation algorithms are selected and compared by simulation tests to demonstrate their advantage and disadvantage. They are: 1. The zero-crossing (ZC) method with linear interpolation; 2. The smart DFT (SDFT) method from [68]; 3. The decomposition method (SDC) from [59]; 4. The signal demodulation (SDM) method [11]. The details of these four algorithms are given in the Appendix. For the SDM, a 6-order Chebyshev type II filter is used to achieve more than 100 dB attenuation for high frequency component with reasonable filter delay. MATLAB is used to implement the algorithms and to generate test signals to disclose different aspects of each algorithm. For all the discrete test signals, the sampling rate is fixed at 3840Hz. To make fair comparison, the additional pre-filters or post-filters are not used.

A. Stationary Signal with Off-nominal Frequencies

In power system, a voltage signal under system steady state is close to a stationary signal that frequency and amplitude are constants. Using the stationary signals with off-nominal frequencies, the basic performance of each algorithm can be disclosed. A number of test signals are produced by

\[ v(t) = A \sin(2\pi ft + 0.3), \]  

where A is a constant and \( f = 61.5\text{Hz}, 59.3\text{Hz}, 58.1\text{Hz}, 45.2\text{Hz}, 20.3\text{Hz} \). Excluding the initial response, the maximum error of each algorithm is given in Table I. The tests demonstrate that all the selected algorithms have the potential to achieve high accuracy for frequency relaying. The ZC, SDFT and SDC have wide range for frequency metering without compromising the accuracy. The range of SDM is limited by the lowpass filter characteristic. The maximum errors for SDFT and SDC are almost zero, because the leakage error for SDFT and the filter error for SDC are completely canceled out. The error of ZC is mainly from the zero-crossing detection, which is improved by linear interpolation. The error from SDM is less than 1mHz if the frequency deviation is less than 10Hz.

B. Tracking the frequency change

During normal operation of the power system, the frequency follows the load / generation variations and fluctuates around nominal value in a range of about ±0.02Hz. When there is major deficit of active power in the system, the frequency would drop at a rate determined by the power unbalance and the system spinning reserve. For system protection and unit protection, it is desirable that frequency change can be detected promptly and accurately. To demonstrate the frequency tracking capability, a few test signals with time-varying frequencies are produced and tested.

1) Signal with time-varying frequency: A similar equation as Eq. (15) is used to produce the test signals with variable frequency. To simulate the voltage signal under load / generation unbalance as the consequence of major generating unit loss, and to reflect the oscillating characteristic of frequency change, the frequency is modified by

\[ f(t) = 57 + 2(1 + 0.4e^{-t} \cos(1.5t - 0.1)) + 0.2e^{-7t/10} \cos(12t). \]

Fig. 6 presents the frequency tracking results of the four algorithms. The dash line in each figure represents the actual time-varying frequency and the solid line gives the frequency tracking results. The ZC, SDFT and SDC demonstrate better dynamic accuracy than SDM. In this test, ZC uses half a cycle to update the frequency so that the estimation delay is small. For an actual application, more cycles are needed for ZC. The latency of SDM is from the lowpass filter delay, which also has obvious effect on the initializing stage when signal is applied. The maximum dynamic error caused by estimation latency is less than 0.08Hz for SDM and less than 0.04Hz for ZC, SDFT and SDC. This test case shows that all these algorithms are performing well with time-varying frequency.

<table>
<thead>
<tr>
<th>( f )</th>
<th>ZC</th>
<th>SDFT</th>
<th>SDC</th>
<th>SDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>61.5Hz</td>
<td>0.1mHz</td>
<td>3E-9mHz</td>
<td>2E-10mHz</td>
<td>0.72mHz</td>
</tr>
<tr>
<td>59.3Hz</td>
<td>0.13mHz</td>
<td>3E-9mHz</td>
<td>2E-10mHz</td>
<td>0.49mHz</td>
</tr>
<tr>
<td>58.1Hz</td>
<td>0.04mHz</td>
<td>7E-9mHz</td>
<td>3E-10mHz</td>
<td>0.35mHz</td>
</tr>
<tr>
<td>45.2Hz</td>
<td>0.04mHz</td>
<td>7E-9mHz</td>
<td>3E-10mHz</td>
<td>2.27mHz</td>
</tr>
<tr>
<td>20.3Hz</td>
<td>2E-3mHz</td>
<td>4.2E-8mHz</td>
<td>3E-9mHz</td>
<td>0.060Hz</td>
</tr>
</tbody>
</table>
2) Both frequency and amplitude are time-varying: In addition to the frequency dynamics, the dynamics of signal amplitude could also have impact on a frequency estimation algorithm. In power system, the voltage signal is generally used for frequency estimation since it is more stable than the current signal. But there are cases that only current signals are available, such as line differential relays or bus differential relays in some substations. What’s more, when the system is experiencing asynchronous oscillations, the voltage and the current would oscillate in both frequency and amplitude. To verify the performance of frequency estimation algorithms under power swing conditions, a signal that is time-varying in both frequency and amplitude is produced per following equations on the basis of Eq. (15),

\[ f(t) = 59.5 + \sin(2\pi t), \quad A(t) = \sqrt{2} + 0.3\cos(3\pi t), \]

where the amplitude is modulated by a 1.5Hz swing and the frequency is modulated by a swing of 1.0Hz. The simulation results of the four algorithms are shown in Fig. 7. In principle, ZC is not affected by the variation of signal amplitude. For SDFT, the impact of the time-varying amplitude is minor since three successive phasors used for each round of estimation will have similar magnitudes under high sampling rate. SDM still has obvious latency in frequency tracking, but it is not caused by the amplitude variations. More dynamic error occurs for SDC because the algorithm assumes that sample values are all the same in the data window.

3) The frequency step test: The step response test is useful to disclose the characteristics of a signal processing algorithm on how it responds to signal changes in time domain. However, the step test for power system frequency has no correspondence in real life. Due to the mass inertia of the rotating machines in the system, it is impossible for the system frequency to have any significant step change. Therefore, an analytical signal with 0.5Hz step change is more than enough to test the frequency estimation algorithms. Fig. 8 gives the measured frequency from four algorithms in response to the MATLAB test signal that changes from 60Hz to 59.5Hz within one sampling interval. The transition period for the ZC, SDF and SDC are within 2 cycles while it takes about 10 cycles for SDM to settle down. The slow response of SDM demonstrate the impact of the lowpass filter that SDM is relying on.

C. Signal containing harmonics, noise and dc component

For a power system signal, the odd number harmonics such as 3rd, 5th, 7th.. harmonics are most likely to occur due to widely-used power electronics and nonlinear load. For system with series-compensated capacitors, the signal may contain low frequency component due to subsynchronous resonance. What’s more, the signal could be contaminated by noise originated from system faults, switching operations, or the electronic circuits. To compare the selected algorithms, a number of discrete signals are produced to simulate extreme conditions in a power system.

1) Signal containing 3rd, 5th, 7th harmonics: A signal with 3rd, 5th, 7th harmonics are produced per following equation,

\[ v(t) = \sqrt{2}\sin(2\pi ft + 0.3) + 0.05\sqrt{2}\sin(6\pi ft) + 0.05\sqrt{2}\sin(10\pi ft) + 0.05\sqrt{2}\sin(14\pi ft), \]

where \( f = 59.5 \). In the equation, the magnitude of fundamental frequency component is 1.0 p.u.. The percentage of 3rd, 5th, 7th harmonics are 5% each. The test results are shown in Fig. 9. The performance of SDM is the best among the four, simply because of the lowpass filter. ZC is less susceptible to harmonics since the zero-crossings on the time axis are mainly determined by the fundamental frequency component, even though the signal is distorted by harmonics. SDFT is susceptible to harmonics because the leakage error of DFT from high frequency components are not compensated. One solution is to add harmonics in the signal model [69] and to compensate the leakage error with the same technique as for fundamental frequency. Another solution is to use a lowpass filter to pre-process the signal and/or to use a moving average filter to smooth the results.

![Fig. 6. Track the frequency that drops dynamically, using: (a) ZC; (b) SDF; (c) SDC; (d) SDM.](image-url)

![Fig. 7. Track the frequency when both amplitude and frequency are time-varying, using: (a) ZC; (b) SDF; (c) SDC; (d) SDM.](image-url)
SDC is better than SDFT, but harmonics still make significant difference for it because the signal model and the frequency equation in SDC are all based on the fundamental frequency component.

2) Signal containing low frequency component: In power system, the interaction between the turbine-generators and series capacitor banks or static VAR control system could cause subsynchronous resonance that introduces low frequency component into the voltage signal for frequency measurement. The low frequency component ranged from 10Hz to 45Hz could last long enough to cause problem to a frequency relay. To investigate the response to different algorithms, a signal with 10% of 25Hz components is produced per following equation.

\[ v(t) = \sqrt{2} \sin(2\pi ft + 0.3) + 0.1\sqrt{2} \sin\left(\frac{25}{30}\pi ft\right) \]

Part of the test signal is shown in Fig. 10 and the test results are shown in Fig. 11. It turns out that subsynchronous resonance could have serious impact on all the frequency estimation algorithms. SDM is performing the best relatively while the errors from ZC, SDFT and SDC are unacceptable. One solution is to detect the low frequency component by DFT and use a notch filter to remove it. However, the DFT window has to be long enough to spot the low frequency component. Another solution is to design a frequency security logic to ignore large frequency variations within a few cycles.

\[ v(t) = 0.5e^{-t/0.3} + \sqrt{2} \sin(2\pi ft + \pi/6). \]

The test results shown in Fig. 12 indicate that DC component can have significant impact on ZC, because the time interval of zero-crossings are changed by DC component. The SDM, SDFT and SDC all use the signal waveform to derive the frequency so that the impact of DC component is less. In most applications, a bandpass filter is necessary to remove the DC component at the price of extra delay.

4) Signal containing impulsive noise: The voltage signal for frequency measurement could be contaminated by impulsive noise or white noise. By changing the randomly-selected sample values, a test signal with impulsive noise is produced to test the frequency measurement algorithms. Fig. 13 presents a portion of the signal. The frequency measurement results of the four algorithms are shown in Fig. 14. The impulsive noise will have adverse impact to all the algorithms. SDM is comparatively better but the results
are still unacceptable. For ZC, the impulsive noise only has effect when it is around zero-crossings. To resolve the problem, either the impulsive noise shall be removed at the pre-processing stage, or the singular frequency estimates can be discarded at the post-processing stage.

5) **Signal containing white noise**: White noise could be introduced by the electromagnetic interference or the deterioration of the electronic components. A test signal with white noise is produced by

\[ v(t) = A \sin(2\pi ft + 0.3) + \varepsilon \]

The parameter \( \varepsilon \) represents the noise that can be produced by a random function on MATLAB. The signal to noise ratio is \( \text{SNR} = 20 \log(1/0.01) = 40\text{dB} \). The test results are shown in Fig. 15. Again, SDM demonstrate its strong anti-noise capability, which attributes to the high attenuation of the lowpass filter for SDM. ZC is relatively less susceptible to white noise. For SDFT and SDC, additional filters must be applied to reduce the error caused by random noise.

### D. Signal from power system simulation

The above test cases are based on analytical signals to disclose different aspects of the selected algorithms for frequency estimation. To simulate a real system, a test signal is generated from a simulation model as shown in Fig. 16. In this two-source model, one of the sources is a 200MVA synchronous machine that is controlled by hydraulic turbine, governor and excitation system. The other source is a simplified voltage source with short circuit capacity 1500MVA.

The system is initialized to start in a steady-state with the generator supplying 200MW of active power to the load. After 0.55s, the breaker that connects the main load to the system is tripped. Because of the sudden loss of load and the inertia of the prime mover, the generator internal voltage starts to oscillate until the control system damps the oscillations.
oscillations. The frequency tracking results from the four algorithms are shown in Fig. 17. The rotor speed is shown in (a) and the frequency tracking from the four algorithms are plotted in (b)-(d). Before the breaker is tripped, the voltage signal is stable and the measured frequency from each algorithm is exactly 60.0Hz. After the breaker is tripped, both the voltage signal and the current signal start to oscillate. Since the voltage is measured at generator terminal, the frequency change should reflect the change of the rotor speed. From Fig. 17, all the four algorithm can tracking the frequency change that is in line with the rotor speed. There is a frequency jump from every estimation algorithm after the breaker is tripped. This drastic frequency variation is caused by the phase abnormality of the signal at the moment of switching operation and should not be accounted as the actual node frequency change. Comparing the four algorithms, SDM gives the most stable and smooth frequency tracking results. However, its recovery from the abnormal frequency jump is also the slowest. The results from SDFT is the worst simply because it is highly susceptible to the harmonics and noise contained in the signal.

VIII. FREQUENCY RELAY DESIGN AND TEST

From the simulation tests, a frequency estimation algorithm alone is not enough to meet the practical requirements for frequency relaying. In order to obtain stable and accurate frequency measurement, it is necessary to add digital filters and security conditions to process the signal and the frequency estimates. Consequently, latency will be introduced to the frequency measurement because of filtering delay and estimation delay. A critical aspect for frequency relay design is to achieve the balance between the accuracy and the group delay, under the condition of robustness. This section discusses a few practical issues about the design and test for frequency relaying.

A. The filtering and post-processing

For a digital frequency relay, the analog anti-alias filter is generally applied to remove the out-of-band components before the A/D conversion (ADC). After ADC, it is necessary to add a digital band-pass filter to remove the harmonics and dc component. For a 60Hz system, the limiting frequencies of the filter passband can be 20 – 65Hz. The stopband attenuation should be as high as possible. However, the filter delay could also become significant for excessive stopband attenuation. To compromise with the filter delay, the average stopband attenuation can be specified at 20 – 40dB, which means that dc component and harmonics are suppressed to 1 – 10% in average. If the filter stopband has valleys corresponding to high attenuation, it is also desirable that the valleys shall be close to the harmonics.

A band-pass filter can effectively remove dc component and harmonics, and helps to reduce white noise to a certain degree. But it cannot handle the impulsive noise. One solution for impulsive noise is to use security conditions at the post-processing stage. Another solution is to use an impulsive noise detector at the pre-processing stage. The detector can determine the impulsive noise as singular sample values according to the adjacent samples. Once detected, the contaminated sample can be replaced by a value that is close to the adjacent samples.

After the frequency estimation, a post-filter would be helpful to get better accuracy, especially when the measured frequency has minor oscillations around the actual frequency. In [39], a 80-coefficient Hamming type FIR filter is applied for post-filtering. In [33], a binomial filter is applied. In many cases, a simple moving average filter is sufficient to improve the measurement accuracy. As a matter of fact, a moving average filter is the optimal filter that can reduce random white noise while keeping sharp step response [56]. However, the length of the moving average filter needs to be carefully selected to achieve the balance between the accuracy and dynamic response of the overall process.

If the signal is distorted under conditions such as CVT transients, CT saturation, system disturbance, switching operations or subsynchronous resonance, erroneous frequency estimates may still exist after the pre-filtering and post-filtering because the filters may not handle all the signal abnormalities. Hence, it is important to have some security conditions to validate the frequency estimation. For example, the difference between two consecutive estimates should be small enough to accept the new estimates; the change of a few consecutive estimates should be consistent, etc. These conditions are based on the fact that power system frequency cannot have drastic change during a few sampling intervals. The security check should also reject the estimates for the first few cycles after the input signal is applied to the relay, because a numerical algorithm needs a few cycles of data to stabilize the estimates.
B. The df/dt measurement

The frequency rate-of-change (df/dt) is a second criteria in a load shedding scheme or remedy action scheme to supervise or accelerate the load shedding. After the frequency is estimated, its rate-of-change is simply computed by the frequency difference and the sampling interval \( \Delta t \),

\[
\frac{df}{dt} = \frac{f(t) - f(t - 1)}{\Delta t}.
\]

This equation could amplify the error or the high frequency component that are contained in the estimated frequency. Hence a lowpass filter and/or a moving average filter are necessary to filter the df/dt outputs. After the filtering, some security conditions similar to those for frequency estimation shall be used to remove abnormal df/dt values.

C. Test results of a frequency relay

Two test cases are presented in this section to show the frequency tracking of an actual relay that is based on zero-crossing principle. The relay is able to achieve 1mHz accuracy for steady state signals and can track the frequency in a large range. To verify its performance under dynamic conditions, the first test signal is produced per following equations,

\[
v(t) = 20e^{-t/0.5} + A(t) \sin(2\pi f(t)t + 0.3) + 0.05A(t) \sin(6\pi f(t)t) + 0.05A(t) \sin(10\pi f(t)t) + 0.05\varepsilon_w(t) + \varepsilon_p(t),
\]

where

\[
f(t) = 60.0 + \sin(2\pi t), \quad A(t) = 40 + 10\cos(3\pi t).
\]

From this equation, the signal contains dc component, harmonics, white noise \( \varepsilon_w \) and impulsive noise \( \varepsilon_p \). The harmonics are 3rd, 5th, 7th at 5% each and the SNR of the white noise is 40dB. In addition, both the frequency \( f(t) \) and amplitude \( A(t) \) are time-varying. This signal represents the extreme condition that is designed to challenge the relay performance. MATLAB is used to create the signal and to save it in a comtrade file, which is then played back to the relay by a real time digital simulator (RTDS). The following test results are taken from the relay’s disturbance recorder that is triggered by the pickup signal of overfrequency element. From the test results in Fig. 18., the impact of dc component, harmonics and noise are reflected by the delay and error in frequency tracking. Compared with the previous simulation test results, the relay has no drastic frequency change or abnormal frequency values during the tracking process, which attributes to the filters and the security check conditions for frequency estimation.

A second signal is produced by the power system simulation model as shown in Fig. 16. Both the frequency and amplitude of the signals oscillate due to the power swing. Compared with the simulation results in Fig. 17, the test results in Fig. 19 are more stable. At the moment when the breaker is tripped, there is no sudden jump in tracking frequency and the frequency is stable throughout the power swing. It shows that the relay is able to handle phase abnormalities, frequency / amplitude oscillations, harmonics and noise, which satisfy the robustness requirement.

D. The test recommendations

To test a frequency relay, the main purpose is to verify the accuracy of frequency measurement and the operating time. The accuracy and operating time under different signal conditions should be tested to verify the robustness of the relay. The test signals need to be practical to reflect the power system operation conditions. For example, a step test that has over 0.5Hz step change may not be appropriate since the step change would not happen to the actual system frequency due to the inertia of rotating machines. The following test items are recommended:

1) Inject signal with different off-nominal frequencies. This test is to verify the steady state accuracy and the measurement range. The accuracy should be consistent for different off-nominal frequencies;

2) Inject signal with ramping-down frequency, the ramping step could be 0.1Hz and the ramping rate could be up to 5Hz/s. If possible, the injected signal, the measured frequency and the frequency rate-of-change shall be recorded in combtrade files for further analysis;

3) Use the same ramping test to check the operating time of the frequency relay when the signal frequency is under/over the setting. The ramping rate can be varied in a reasonable range (such as 0.5Hz - 5Hz) to check the relay response;

4) Inject the relay with contaminated signal. The 3rd, 5rd, 7th harmonics that are 5% each can be added.
onto the main signal. The dc component and random noise should also be added if possible; Otherwise, the playback test can be used instead.

5) If the test equipment supports playback test, a number of comtrade files with simulated test signals can be created and played back to the frequency relay. MATLAB or MATHCAD could be used to produce analytical signals per equations in this paper. The transient simulation software such as EMTP can also be used to produced signals by using power system models. The tracking frequency and frequency rate-of-change can be recorded in comtrade files to deduce the maximum error, average error and the response time. These tests should be designed to verify the robustness of the frequency relay under adverse conditions.

During the tests, it should be noted that a frequency relay needs a few cycles to stabilize at the beginning of the injection test. So it is recommended to give the relay at least ten cycles of stable signal before any signal variations. The frequency measurement error during this initialization process should not be accounted.

IX. SUMMARY

As a fundamental characteristic of considered signal, the frequency and its measurement are important not only to under-frequency relays but also to other protective relays. Many numerical algorithms were discussed to pursue both high accuracy and fast response on frequency measurement. However, it should be emphasized that power system frequency is not an instantaneous value, even though the concept of instantaneous frequency could be utilized in some algorithms. Due to the mass inertia of rotating machines, the system frequency cannot have step change or fast change. It is justifiable to use a window of data to compute the average frequency to approximate the system frequency. A few cycles delay is not only allowed but necessary for robust frequency relaying. The accuracy of frequency measurement only makes sense when the signal conditions are specified. If the signal is contaminated or distorted, it is more important to maintain a stable frequency measurement instead of pursuing fast response. To design a frequency relay, the digital filters and security check conditions should be applied to avoid abnormal frequency output under various conditions, so that the frequency relay or other protective relays can securely protect the machines and the system.

X. APPENDIX

A. Zero-crossing with linear interpolation

For zero-crossing method implemented numerically, it is important to detect the zero-crossings accurately on the time axis. The linear interpolation method can be applied to serve this purpose. A zero-crossing is found as between two neighboring samples with different signs. The crossing point of the line that connects the two samples and the time axis is taken as the zero-crossing point. The line is expressed as

\[ p(x) = (x - m + 1)v_m - (x - m)v_{m-1}. \]

where \( v_{m-1} \) and \( v_m \) are sample values, \( x \) and \( m \) are sample indexes. Let \( p(x) = 0 \), the zero-crossing in term of sample index is obtained by

\[ x = m - \frac{v_m}{v_m - v_{m-1}}. \]

Though it is still called sample index, \( x \) is usually a fractional value between \( m - 1 \) and \( m \). Let \( T_r \) represent the sampling interval, the zero-crossing is \( t = xT_r \). The frequency is calculated from two consecutive zero-crossings at \( t_1 \) and \( t_2 \),

\[ f = \frac{1}{2(t_1 - t_2)} = \frac{1}{2T_r(x_1 - x_2)}. \]

B. Yang&Liu’s smart DFT method (SDFT)

In [68], the phasor and frequency are derived from a compensated DFT method that leakage error can be completely canceled out. The signal model

\[ v(t) = V \cos(2\pi(f_0 + \Delta f)t + \varphi) \]

is firstly written as

\[ v(t) = \frac{ve^{j2\pi(f_0 + \Delta f)t} + ve^{-j2\pi(f_0 + \Delta f)t}}{2} \]

where \( v = ve^{j\varphi} \) is the phasor, \( f_0 \) is the nominal frequency and \( \Delta f \) represents the frequency deviation. Applying DFT to \( v(t) \), the estimation of fundamental frequency component is

\[ \hat{v}_r = \frac{2}{N} \sum_{k=0}^{N-1} v(k + r)e^{-j\frac{2\pi k}{N}} \]

where the subscript \( r \) is the index of the first sample in the DFT window. Discretizing Eq. (18) by \( t = \frac{k}{f_0N} \) and bring it into Eq. (19), the estimated phasor is expressed by

\[ \hat{v}_r = A_r + B_r \]

where

\[ A_r = v \sin \frac{N\theta_1}{N} \sin \frac{N\theta_2}{2} \exp \left[ j \frac{\pi}{f_0N} (\Delta f(2r + N - 1) + 2f_0r) \right], \]

\[ B_r = v^* \sin \frac{N\theta_2}{2} \exp \left[ -j \frac{\pi}{f_0N} (\Delta f(2r + N - 1) + 2f_0r + N - 1) \right], \]

\[ \theta_1 = \frac{2\pi\Delta f}{f_0N}, \quad \theta_2 = \frac{2\pi(2 + \frac{4\Delta f}{f_0})}{N}. \]

Applying DFT onto two consecutive data windows, the following relationship holds,

\[ A_{r+1} = A_r a, \quad B_{r+1} = B_r a^{-1} \]

where

\[ a = \exp \left[ j \frac{2\pi(f_0 + \Delta f)}{f_0N} \right]. \]
From Eq. (20) and Eq. (21), there are
\[ \hat{v}_{r+1} = A_r * a + B_r * a^{-1} \]
\[ \hat{v}_{r+2} = A_r * a^2 + B_r * a^{-2}. \] (23) (24)
Combining Eq. (20)-(24), the \( A_r \) and \( B_r \) can be eliminated and a 2nd-order polynomial is formed as
\[ \hat{v}_{r+1} * a^2 - (\hat{v}_r + \hat{v}_{r+2}) * a + \hat{v}_{r+1} = 0. \]
The root is
\[ a = \frac{(\hat{v} + \hat{v}_{r+2}) \pm \sqrt{(\hat{v} + \hat{v}_{r+2})^2 - 4\hat{v}_{r+1}^2}}{2\hat{v}_{r+1}}. \]
From Eq. (21), the frequency is calculated by
\[ f = f_0 + \Delta f = \frac{f_0 N}{2\pi} \cos^{-1}(\text{Re}(a)). \]
This method does not make any approximation to derive the frequency. The leakage error is canceled out from three consecutive phasors so that the algorithm is highly accurate for stable signals and can follow the frequency change promptly.

### C. Szafran’s signal decomposition method (SDC)

In [59], the orthogonal decomposition method is modified to eliminate the error caused by unequal filter gains at off-nominal frequencies. The signal is firstly decomposed by a pair of orthogonal FIR filters, such as a sine filter and a cosine filters. For an input sinusoidal signal in discrete form, the outputs \( y_c(n) \) and \( y_s(n) \) of the filters are
\[ y_c(n) = |F_c(\omega)| A \cos(n\omega T_s + \varphi + \alpha(\omega)), \]
\[ y_s(n) = |F_s(\omega)| A \sin(n\omega T_s + \varphi + \alpha(\omega)), \]
where \( |F_c(\omega)| \) and \( |F_s(\omega)| \) are the filter gains at frequency \( \omega \). A new signal \( g_k(\omega) \) can be composed by using the historical output signals \( y_c(n-k) \) and \( y_s(n-k) \),
\[ g_k(\omega) = y_c(n)y_s(n-k) - y_s(n)y_c(n-k), \] (25)
Again, by utilizing the historical signal \( g_{2k}(\omega) \), an expression independent of signal magnitude can be obtained,
\[ \frac{g_{2k}(\omega)}{g_k(\omega)} = 2 \cos(k\omega T_s). \] (26)
Bring Eq. (25) into Eq. (26), the frequency is calculated by
\[ f = \frac{1}{2\pi kT_s} \cos^{-1} \left( \frac{1}{2} \frac{y_s(n)y_c(n-2k) - y_c(n)y_s(n-2k)}{y_s(n)y_c(n-k) - y_c(n)y_s(n-k)} \right). \] (27)
Since the error incurred by orthogonal filters are canceled out in Eq. (26), high accuracy can be achieved.

### D. Signal demodulation Method (SDM)

Using the simple signal model in Eq. (8), a new signal \( Y(t) \) is generated by multiplying \( v(t) \) with a reference signal that has nominal frequency \( \omega_0 \),
\[ Y(t) = v(t)e^{-j\omega_0 t} = \frac{A}{2} e^{j((\omega - \omega_0)t + \varphi)} + \frac{A}{2} e^{-j((\omega + \omega_0)t + \varphi)}. \] (28)
The signal \( Y(t) \) has a low frequency component and a near-double frequency \( (\omega + \omega_0) \) component. A low-pass filter can be applied to filter the near-double frequency component so that the remaining signal \( y(t) \) contains the frequency deviation information. Using discrete form of \( y(t) \), another complex signal \( U(k) \) is produced by
\[ U(k) = y(k)y^*(k-1), \]
where \( y^*(k-1) \) is the conjugate of \( y(k-1) \). The frequency is derived by
\[ \hat{f}(k) = f_0 + \frac{f_2}{2\pi} \tan^{-1} \left( \frac{\text{Im}(U(k))}{\text{Re}(U(k))} \right). \]

### E. The supervised Gauss-Newton method

The supervised Gauss-Newton (SGN) method is based on [60] with two additional auxiliary algorithms, ZC and DFT, to supervise the Gauss-Newton process. The ZC and DFT are used to estimate the frequency and signal amplitude roughly so that the parameters can be properly initialized in each iteration.

The Gauss-Newton process is an iterative method to estimate the model parameters by minimizing the error between the estimation and the observation. Let the parameter vector be \( x \) and the objective function be \( f(x) \). Starting from a initial point \( x_0 \), if the descending condition \( |f(x_{k+1})| < |f(x_k)| \) could be enforced, a series of vectors \( x_1, x_2, \ldots \) could be iteratively calculated and \( x \) will finally converge to \( x^* \), a minimizer of the objective function \( f(x) \). In SGN, the parameter vector \( x \) is selected per basic signal model in Eq. (8),
\[ x = [A(t) \quad \omega(t) \quad \varphi(t)]^T. \] (29)
The process of parameters updating is expressed by
\[ x = x + \Delta x. \] (30)
The objective function \( f(x) \) is defined as the error between the estimated signal value \( v_{est}(x) \) from Eq. (8) and the sample value \( v_{obs} \),
\[ f(x) = v_{est}(x) - v_{obs}. \]
The Gauss-Newton updating step is expressed by
\[ \Delta x = (J^TJ)^{-1}J^Tf(x). \] (31)
where \( J \) is the Jacobian matrix containing partial derivatives for the estimated parameters. The Eq. (31) will maintain the decent direction to minimize the objective function \( f(x) \). In order to apply the algorithm in a real-time application, it is necessary to stop the iterations when the error function
\( f(x) \) is close to zero, or when the gradient function \( J^T f(x) \) is close to zero. It can be proven that Gauss-Newton method can only achieve linear convergence when \( x \) is far from the minimizer \( x^* \) while quadratic convergence is possible when \( x \) is close to \( x^* \). If the initial \( x \) has significant difference from \( x^* \), the convergence may not even be achieved. In order to get proper initial parameters and fast convergence, two auxiliary algorithms can be used to provide initial parameters and to supervise each Gauss-Newton updating step. The recursive DFT method is used to estimate the signal amplitude and the zero-crossing (ZC) method is used to estimate the frequency. The results of DFT and ZC will be close to the actual value but not necessarily be accurate. With this combined approach, fast convergence can be achieved.

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